

Identities in Nonlinear Realizations of Supersymmetry

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Abstract

In this paper, we emphasize that a UV SUSY-breaking theory can be realized either linearly or nonlinearly. Both realizations form the dual descriptions of the UV SUSY-breaking theory. Guided by this observation, we find subtle identities involving the Goldstino field and matter fields in the standard nonlinear realization from trivial ones in the linear realization. Rather complicated integrands in the standard nonlinear realization are identified as total-divergences. Especially, identities only involving the Goldstino field reveal the self-consistency of the Grassmann algebra. As an application of these identities, we prove that the nonlinear Kahler potential without or with gauge interactions is unique, if the corresponding linear one is fixed. Our identities pick out the total-divergence terms and guarantee this uniqueness.

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As a possible extension of the Standard Model, supersymmetry (SUSY) has attracted much attention over several decades. SUSY has the important feature that it provides a reasonable framework to circumvent the hierarchy problem. One may start the discussion of SUSY theories from the SUSY algebra, which is the only viable graded Lie algebra consistent with other requirements for a non-trivial relativistic quantum field theory. For any field A , the N=1 SUSY algebra requires (for review, see [1],[2],[3])

$$(\delta_\eta \delta_\xi - \delta_\xi \delta_\eta) A = -2i (\eta \sigma^\mu \bar{\xi} - \xi \sigma^\mu \bar{\eta}) \partial_\mu A, \quad (1)$$

where the infinitesimal SUSY transformation δ_ξ is realized by the operator $\xi Q + \bar{\xi} \bar{Q}$ as

$$\delta_\xi A = (\xi Q + \bar{\xi} \bar{Q}) A. \quad (2)$$

Much has been discussed for SUSY models and phenomenological implications. Hopefully, one is to see some SUSY signals in the running LHC experiment.

In order to be consistent with existing experimental measurements and to be of phenomenological relevance, SUSY must be broken and broken spontaneously at the TeV scale. According to the general theory of spontaneously symmetry breaking, this should lead to a massless neutral Nambu-Goldstone fermion, the Goldstino field. To deal with the low energy SUSY physics, it is expedient to study fields in the framework of nonlinear realization of SUSY. In the nonlinear realization, vertices with Goldstino fields always carry at least one space-time derivative, as one would have expected. So it is difficult to directly pick out the total-divergence terms in the nonlinear actions. In the Appendix 2 of [4], a rather complicated integrand related to Goldstino field was proven to be total-divergence via tedious Fierz rearrangements. This total-divergence term simplifies the non-minimal contribution of the Goldstino action. This feature brings us to study how to identify more total-divergence terms in the nonlinear SUSY actions.

In this paper, we observe more identities in the nonlinear SUSY actions. We emphasize that a UV SUSY-breaking theory could be realized either linearly or nonlinearly. Both realizations form dual descriptions of the same UV SUSY-breaking theory. Keeping the dual description in mind, the total-divergence terms in a linear SUSY theory can be transformed into their nonlinear versions. We thus obtain intriguing identities about the Goldstino field and matter fields in the standard nonlinear realization. Rather complicated integrands are identified as total-divergences. One of these identities is exactly the one in the Appendix

2 of [4] and other identities are first obtained. Moreover, we show that the identity in the Appendix 2 of [4] reveals the self-consistency of the Grassmann algebra. As an application of our identities, we prove presently the equivalence between some different kinds of nonlinear Kahler potentials. The nonlinear Kahler potential without or with gauge interactions is unique, if the corresponding linear one is fixed. Our identities pick out the total-divergence terms and guarantee this uniqueness.

Primarily, we will discuss the standard nonlinear realization as first introduced in [5]. SUSY transformations of the Goldstino field λ and the matter field ζ are as follows [1]

$$\begin{cases} \delta_\xi \lambda_\alpha &= \frac{\xi_\alpha}{\kappa} - i\kappa (\lambda \sigma^\mu \bar{\xi} - \xi \sigma^\mu \bar{\lambda}) \partial_\mu \lambda_\alpha, \\ \delta_\xi \zeta &= -i\kappa (\lambda \sigma^\mu \bar{\xi} - \xi \sigma^\mu \bar{\lambda}) \partial_\mu \zeta. \end{cases} \quad (3)$$

It is easy to check that both equations in Eq (3) satisfy the closure relation Eq (1). This nonlinear realization could be constructed from the linear one with the help of the standard realization of the nonlinear SUSY [4],[6],[7]. It is based upon the following observation: with the help of the nonlinear Goldstino field λ , a linear superfield $\hat{\Omega}(x, \theta, \bar{\theta})$ can be converted into a set of nonlinear matter fields via¹

$$\Omega(x, \theta, \bar{\theta}) = \exp [-\kappa \lambda(x) Q - \kappa \bar{\lambda}(x) \bar{Q}] \times \hat{\Omega}(x, \theta, \bar{\theta}), \quad (4)$$

where Q_α and $\bar{Q}_{\dot{\alpha}}$ are generators of SUSY algebra realized in superspace

$$\begin{cases} Q_\alpha = \partial_\alpha + i (\sigma^\mu \bar{\theta})_\alpha \partial_\mu, \\ \bar{Q}_{\dot{\alpha}} = -\partial_{\dot{\alpha}} - i (\theta \sigma^\mu)_{\dot{\alpha}} \partial_\mu. \end{cases} \quad (5)$$

Since λ transforms according to the first equation in Eq (3) and $\hat{\Omega}(x, \theta, \bar{\theta})$ transforms according to Eq (2), it is easy to prove that $\Omega(x, \theta, \bar{\theta})$ transforms according to the second equation in Eq (3). Following these redefinitions of fields, any UV actions in linear SUSY-breaking theories could be re-expressed in the language of nonlinear fields [4],[8]. This enables us to construct dual descriptions of the same UV SUSY-breaking theory, which could be described either linearly or nonlinearly. Both realizations have the same degrees of freedom. The difference are in definitions of fields, which respect the SUSY algebra in their own ways.

¹ In this paper, superfields and their components in the linear SUSY are hatted while their counterparts in the nonlinear SUSY are not. We use the conventions of [1]. All symbols can be found in [8], if not explicitly defined in this paper.

Both realizations can be connected by the so-called standard realization of the nonlinear SUSY [4],[6],[7]. To show this duality, we take the UV canonical Kahler potential as an example. For a chiral superfield $\hat{\Phi}$ in a linear theory, the canonical Kahler potential is $\mathcal{S}_{\text{can}} = \int d^4x d^4\theta \hat{\Phi}^\dagger \hat{\Phi}$. After fields redefinitions², one has $\mathcal{S}_{\text{can}}^{\text{NL}} = \int d^4x d^4\theta \det T \det M \Phi^\dagger \Phi$ in its dual nonlinear description [4],[8]. Both \mathcal{S}_{can} and $\mathcal{S}_{\text{can}}^{\text{NL}}$ have the same degrees of freedom and describe the same physical process.

Interestingly, the canonical Kahler potential in linear theory may be written as $\mathcal{S}'_{\text{can}} = \int d^4x d^4\theta \frac{1}{2} \left(e^{-2i\theta\sigma^\mu\bar{\theta}\partial_\mu} \hat{\varphi}^\dagger \hat{\varphi} + \hat{\varphi}^\dagger e^{2i\theta\sigma^\mu\bar{\theta}\partial_\mu} \hat{\varphi} \right)$ by dropping possible total-divergence terms. Rewriting $\mathcal{S}'_{\text{can}}$ in terms of nonlinear chiral superfields [4],[8], we obtain $\mathcal{S}'_{\text{can}}^{\text{NL}} = \int d^4x d^4\theta \frac{1}{2} \det T \left(\det M_+ e^{-2i\theta\sigma^\mu\bar{\theta}\Delta_\mu^-} \varphi^\dagger \varphi + \det M_- \varphi^\dagger e^{2i\theta\sigma^\mu\bar{\theta}\Delta_\mu^+} \varphi \right)$. No doubt both \mathcal{S}_{can} and $\mathcal{S}'_{\text{can}}$ describe the same physical process. As dual descriptions, the nonlinear versions $\mathcal{S}_{\text{can}}^{\text{NL}}$ and $\mathcal{S}'_{\text{can}}^{\text{NL}}$ should describe the same physics too. However, the integrands of $\mathcal{S}_{\text{can}}^{\text{NL}}$ and $\mathcal{S}'_{\text{can}}^{\text{NL}}$ have quite different terms after integrating out the θ 's. In this paper, we will show that those different terms can be identified as total-divergences. As an application of our identities, we will show the equivalence between $\mathcal{S}_{\text{can}}^{\text{NL}}$ and $\mathcal{S}'_{\text{can}}^{\text{NL}}$ in the end of this paper.

We start our discussion from Grassmann algebra in any linear theory, which is the simplest case in our observations. Grassmann algebra requires the identity $\int d^4x d^2\theta \times 1 = 0$. In its dual description, this can be re-expressed in terms of the nonlinear fields as

$$\int d^4x d^2\theta \det T \det M_+ \times 1 = 0. \quad (6)$$

Here $\det T \det M_+$ is the Jacobian determinant of the transformation Eq (4), with $T_\mu^\nu = \delta_\mu^\nu - i\kappa^2 \partial_\mu \lambda \sigma^\nu \bar{\lambda} + i\kappa^2 \lambda \sigma^\nu \partial_\mu \bar{\lambda}$ and $M_{+\mu}^\nu = \delta_\mu^\nu - 2i\kappa\theta\sigma^\nu \bar{\lambda}_\mu$ [4]. Explicitly,

$$\det T = 1 - i\kappa^2 (\partial_\mu \lambda \sigma^\mu \bar{\lambda} - \lambda \sigma^\mu \partial_\mu \bar{\lambda}) \quad (7)$$

$$- \kappa^4 (i\epsilon^{\mu\nu\rho\gamma} \lambda \sigma_\rho \bar{\lambda} \partial_\mu \lambda \sigma_\gamma \partial_\nu \bar{\lambda} + \bar{\lambda}^2 \partial_\mu \lambda \sigma^{\mu\nu} \partial_\nu \lambda + \lambda^2 \partial_\mu \bar{\lambda} \bar{\sigma}^{\mu\nu} \partial_\nu \bar{\lambda})$$

$$- i\kappa^6 \lambda^2 \bar{\lambda} (\bar{\sigma}^\rho \partial_\rho \lambda \partial_\mu \bar{\lambda} \bar{\sigma}^{\mu\nu} \partial_\nu \bar{\lambda} + 2\bar{\sigma}^\nu \partial_\mu \lambda \partial_\nu \bar{\lambda} \bar{\sigma}^{\rho\mu} \partial_\rho \bar{\lambda})$$

$$- i\kappa^6 \bar{\lambda}^2 \lambda (\sigma^\rho \partial_\rho \bar{\lambda} \partial_\mu \lambda \sigma^{\mu\nu} \partial_\nu \lambda + 2\sigma^\nu \partial_\mu \bar{\lambda} \partial_\nu \lambda \sigma^{\rho\mu} \partial_\rho \lambda),$$

$$\det M_+ = 1 - 2i\kappa\theta\sigma^\mu \bar{\lambda}_\mu + 4\kappa^2\theta^2 \bar{\lambda}_\mu \bar{\sigma}^{\nu\mu} \bar{\lambda}_\nu, \quad (8)$$

² In order to get SUSY-breaking, the F-term of the linear chiral field $\hat{\Phi}$ should develop a non-zero VEV $\langle F \rangle$. The nonlinear Goldstino field can then be constructed from fields of the linear theory by setting the fermionic component vanishing [9].

with $\lambda_\mu = \nabla_\mu \lambda$, $\bar{\lambda}_\mu = \nabla_\mu \bar{\lambda}$ and $\nabla_\mu = (T^{-1})^\nu_\mu \partial_\nu$. Naively, one may expect $\kappa^8 \lambda^2 \bar{\lambda}^2$ terms in $\det T$ by expanding T^ν_μ directly. However, this term vanishes actually [10],[11],[12]. Integrating out the θ 's, one has

$$\int d^4x \det T \bar{\lambda}_\mu \bar{\sigma}^{\nu\mu} \bar{\lambda}_\nu = 0. \quad (9)$$

Starting with $\int d^4x d^2\bar{\theta} \times 1 = 0$, one obtains another identity

$$\int d^4x \det T \lambda_\mu \sigma^{\nu\mu} \lambda_\nu = 0, \quad (10)$$

which is the conjugate of Eq (9).

Eq (10) tells us that the term $\det T \lambda_\mu \sigma^{\nu\mu} \lambda_\nu$ should be a total divergence term. So its integration, which masquerades as a non-zero result, should vanish. In fact, this conclusion was first shown in the Appendix 2 of [4], where $\det T \lambda_\mu \sigma^{\nu\mu} \lambda_\nu$ was proven to be a total divergence term via tedious Fierz rearrangements. We comment that Eq (10) reveals the self-consistency of Grassmann algebra. One starting point of the SUSY nonlinear realization is to identify a Grassmannian coordinate ξ with the Goldstino field λ , namely $\xi \rightarrow \kappa \lambda$ [1],[5]. Eq (4) is essentially a special shift $\theta' = \theta - \kappa \lambda(x)$ in superspace. The shift-invariance of Grassmann algebra demands both $\int d^4x d^2\theta \times 1 = 0$ and $\int d^4x' d^2\theta' \times 1 = 0$, which leads to Eq(9). This integral gives no contribution to any physical process. Its integrand would be discarded when a Lagrangian of the Goldstino field is constructed [4].

Grassmann algebra also requires $\int d^4x d^4\theta \times 1 = 0$ in a linear theory. This can also be re-expressed in the nonlinear realization as

$$\int d^4x d^4\theta \det T \det M \times 1 = 0, \quad (11)$$

where $M^\nu_\mu = \delta^\nu_\mu - i\kappa \theta \sigma^\nu \bar{\lambda}_\mu + i\kappa \lambda_\mu \sigma^\nu \bar{\theta}$ and

$$\begin{aligned} \det M = & 1 + i\kappa (\lambda_\mu \sigma^\mu \bar{\theta} - \theta \sigma^\mu \bar{\lambda}_\mu) \\ & - \kappa^2 (i\epsilon^{\mu\nu\rho\gamma} \theta \sigma_\rho \bar{\theta} \lambda_\mu \sigma_\gamma \bar{\lambda}_\nu + \bar{\theta}^2 \lambda_\mu \sigma^{\mu\nu} \lambda_\nu + \theta^2 \bar{\lambda}_\mu \bar{\sigma}^{\mu\nu} \bar{\lambda}_\nu) \\ & + i\kappa^3 \theta^2 \bar{\theta} (\bar{\sigma}^\rho \lambda_\rho \bar{\lambda}_\mu \bar{\sigma}^{\mu\nu} \bar{\lambda}_\nu + 2\bar{\sigma}^\nu \lambda_\mu \bar{\lambda}_\nu \bar{\sigma}^{\rho\mu} \bar{\lambda}_\rho) \\ & + i\kappa^3 \bar{\theta}^2 \theta (\sigma^\rho \bar{\lambda}_\rho \lambda_\mu \sigma^{\mu\nu} \lambda_\nu + 2\sigma^\nu \bar{\lambda}_\mu \lambda_\nu \sigma^{\rho\mu} \lambda_\rho). \end{aligned} \quad (12)$$

Noticing that there is no $\theta^2 \bar{\theta}^2$ term in $\det M$, just as there is no $\kappa^8 \lambda^2 \bar{\lambda}^2$ term in $\det T$. So Eq (11) leads to a trivial but self-consistent result.

From a linear chiral superfield $\hat{\Phi}(x, \theta, \bar{\theta}) = \exp(i\theta \sigma^\mu \bar{\theta} \partial_\mu) \hat{\varphi}(x, \theta)$ with $\hat{\varphi}(x, \theta) = \hat{\phi} + \sqrt{2}\theta \hat{\psi} + \theta^2 \hat{F}$, a nonlinear Φ can be obtained via Eq (4) as $\Phi(x, \theta, \bar{\theta}) = \exp(i\theta \sigma^\mu \bar{\theta} \Delta_\mu^+) \varphi(x, \theta)$, with

$\varphi(x, \theta) = \phi + \sqrt{2}\theta\psi + \theta^2 F$ and $\Delta_\mu^+ = (M_+^{-1})_\mu{}^\nu (\nabla_\nu + \kappa\lambda_\nu^\alpha \partial_\alpha)$ [4],[8]. Here ϕ , ψ and F are composites of $\hat{\phi}$, $\hat{\psi}$, \hat{F} and λ , whose explicit forms are governed by Eq (4). Starting from a chiral field $\hat{\varphi}$ in the linear theory, one has

$$\int d^4x d^2\theta \partial_\mu \hat{\varphi} = 0. \quad (13)$$

Rewriting this identity in terms of nonlinear fields, we have

$$\int d^4x d^2\theta \det T \det M_+ \Delta_\mu^+ \varphi = 0. \quad (14)$$

Being a matter field in the nonlinear realization, each component field of φ transforms into itself and is independent of other fields. Thus Eq (14) contains actually three independent identities

$$\begin{cases} \int d^4x d^2\theta \det T \det M_+ \Delta_\mu^+ (\phi) = 0, \\ \int d^4x d^2\theta \det T \det M_+ \Delta_\mu^+ (\sqrt{2}\theta\psi) = 0, \\ \int d^4x d^2\theta \det T \det M_+ \Delta_\mu^+ (F\theta^2) = 0. \end{cases} \quad (15)$$

Explicitly, one has

$$\int d^4x \det T 4\kappa^2 (\bar{\lambda}_\rho \bar{\sigma}^{\nu\rho} \bar{\lambda}_\nu \nabla_\mu \phi + 2\bar{\lambda}_\mu \bar{\sigma}^{\nu\rho} \bar{\lambda}_\rho \nabla_\nu \phi) = 0, \quad (16)$$

$$\int d^4x \det T \sqrt{2}\kappa [i(\nabla_\mu \psi \sigma^\nu \bar{\lambda}_\nu - \nabla_\nu \psi \sigma^\nu \bar{\lambda}_\mu) + 4\kappa^2 (\bar{\lambda}_\rho \bar{\sigma}^{\nu\rho} \bar{\lambda}_\nu \lambda_\mu \psi + 2\bar{\lambda}_\mu \bar{\sigma}^{\nu\rho} \bar{\lambda}_\rho \lambda_\nu \psi)] = 0, \quad (17)$$

$$\int d^4x \det T [\nabla_\mu F + 2i\kappa^2 F (\lambda_\mu \sigma^\nu \bar{\lambda}_\nu - \lambda_\nu \sigma^\nu \bar{\lambda}_\mu)] = 0. \quad (18)$$

We now have three complicated integrands which are total-divergences. The corresponding integrals vanish and do not contribute to any physical process. Having obtained these identities, the explicit forms of ϕ , ψ and F are irrelevant. ϕ , ψ and F could be any fundamental or composite fields. From the linear identity $\int d^4x d^2\theta \partial_\mu \hat{\varphi}^\dagger = 0$, three identities about ϕ^\dagger , $\bar{\psi}$ and F^\dagger can be obtained. They are just conjugate identities of Eq (16), Eq(17) and Eq (18), respectively.

More identities can be obtained following the same spirit.³ There should be more identities if one considers a general super-field $\hat{\Phi}$ of nine independent component fields. Nine new

³ Identities with spinor indices in nonlinear theories can be derived from non-linearizing $\int d^4x d^4\theta D_\alpha \hat{\Phi} = 0$ and $\int d^4x d^4\theta \bar{D}_{\dot{\alpha}} \hat{\Phi} = 0$ in linear theories. Since they are not Lorentz scalars, thus of no help to constructing nonlinear actions. We do not discuss them in this paper.

identities result from non-linearizing $\int d^4x d^4\theta \partial_\mu \hat{\Phi} = 0$ and they will be listed in Appendix. As dual descriptions, those processes could be regarded as rewritten identities in the language of the nonlinear fields. These identities are very complicated and hard to be obtained via other methods. They all contain integrands that are total-divergences, where matter fields can be either fundamental or composite. Thus, no new identities can be obtained even if one starts with identities with more than one derivatives in the linear theory, such as $\int d^4x d^4\theta \partial_\mu \partial_\nu \hat{\Phi} = 0$.

In summary, Eq (9) only involves the Goldstino field. A nonlinear complex scalar field ϕ , not matter fundamental or composite, obeys Eq (16), Eq (18), Eq (A7) and Eq (A10). Correspondingly, a nonlinear complex scalar field ϕ^\dagger obeys the conjugate identities of them. A nonlinear complex fermion field ψ , fundamental or composite, obeys Eq (17), Eq (A4) and Eq (A9). Conjugate identities of Eq (17), Eq (A4) and Eq (A9) are concerning a nonlinear complex fermion field $\bar{\psi}$. For a nonlinear vector boson field ν_u , it should obey the same identities as a scalar field should. Moreover, ν_u should obey Eq (A5) especially.

Our dual descriptions are only applicable to UV SUSY-breaking theories, which could be described either linearly or nonlinearly. When some superpartners have masses much higher than the typical energy scale of the concerned physical processes, they could be integrated out. In contrast to the linear realization, heavy particles can be integrated out without breaking SUSY in the nonlinear realization. The low energy spectrum does not consist of complete supersymmetric multiplets, but all remaining fields respect the SUSY algebra in a nonlinear fashion. We cannot construct dual descriptions for low energy effective SUSY-breaking theories. However, our identities are still valid for low energy effective SUSY actions. This is because of the same algebraic structure of the nonlinear SUSY realization. A general nonlinear effective action, which is SUSY and gauge invariant, is given as [13],[14],[15]

$$\mathcal{S}_{\text{eff}} = \int d^4x \det T \mathcal{L}_{\text{eff}}(\lambda_\mu, \bar{\lambda}_\mu, \phi^i, \nabla_\mu \phi^i, \mathcal{F}_{\mu\nu}). \quad (19)$$

Here the effective Lagrangian \mathcal{L}_{eff} can be expanded in the number of the Goldstino fields, namely $\mathcal{L}_{\text{eff}} = \mathcal{L}_{(0)} + \mathcal{L}_{(1)} + \mathcal{L}_{(2)} + \dots$. The leading-order term $\mathcal{L}_{(0)}$ is fixed by the Lagrangian of MSSM while the high-order terms $\mathcal{L}_{(n)}$ are essentially model dependent. To be variant under nonlinear SUSY transformations, effective actions could be constructed order by order in the number of Goldstino fields. In principle our new identities can simplify \mathcal{L}_{eff} by finding the total-divergences, which masquerade as non-zero results. However, none such application

has been seen because it is challenging to construct the most general \mathcal{L}_{eff} via this method.

As an application of these identities, we will prove presently the equivalence between the canonical Kahler potential $\mathcal{S}_{\text{can}}^{\text{NL}}$ and $\mathcal{S}_{\text{can}}^{\prime\text{NL}}$. Keeping the dual description in mind, $\mathcal{S}_{\text{can}}^{\text{NL}}$ and $\mathcal{S}_{\text{can}}^{\prime\text{NL}}$ should describe the same physics, since their linear versions \mathcal{S}_{can} and $\mathcal{S}_{\text{can}}'$ do. As discussed before, $\mathcal{S}_{\text{can}}^{\text{NL}} = \int d^4x d^4\theta \det T \det M \Phi^\dagger \Phi$ while $\mathcal{S}_{\text{can}}^{\prime\text{NL}} = \int d^4x d^4\theta \frac{1}{2} \det T \left(\det M_+ e^{-2i\theta\sigma^\mu \bar{\theta} \Delta_\mu^-} \varphi^\dagger \varphi + \det M_- \varphi^\dagger e^{2i\theta\sigma^\mu \bar{\theta} \Delta_\mu^+} \varphi \right)$ in the nonlinear realization. Integrating out the θ 's, one has

$$\begin{aligned} \mathcal{S}_{\text{can}}^{\text{NL}} = \int d^4x \det T \Bigg[& F^\dagger F + \frac{1}{2} \phi \nabla^\mu \phi_\mu^\dagger + \frac{1}{2} \phi^\dagger \nabla^\mu \phi_\mu - \frac{1}{4} \nabla_\mu (\phi \phi^{\mu\dagger}) - \frac{1}{4} \nabla_\mu (\phi^\dagger \phi^\mu) \\ & + \frac{i}{2} \nabla_\mu \bar{\psi} \bar{\sigma}^\mu \psi - \frac{i}{2} \bar{\psi} \bar{\sigma}^\mu \nabla_\mu \psi + \frac{\kappa}{\sqrt{2}} \phi_\mu^\dagger \lambda_\nu \sigma^\mu \bar{\sigma}^\nu \psi + \frac{\kappa}{\sqrt{2}} \phi_\mu \bar{\lambda}_\nu \bar{\sigma}^\mu \sigma^\nu \bar{\psi} \\ & + \frac{\kappa^2}{2} \epsilon^{\mu\nu\rho\gamma} \lambda_\mu \sigma_\gamma \bar{\lambda}_\nu (\phi \phi_\rho^\dagger - \phi^\dagger \phi_\rho) - i\kappa^2 \phi \phi_\mu^\dagger \lambda_\nu \sigma^\mu \bar{\sigma}^{\nu\rho} \bar{\lambda}_\rho + i\kappa^2 \phi^\dagger \phi_\mu \lambda_\rho \sigma^{\rho\nu} \sigma^\mu \bar{\lambda}_\nu \\ & + \frac{\kappa}{\sqrt{2}} \bar{\lambda}_\mu \bar{\sigma}^{\mu\nu} \nabla_\nu \bar{\psi} \phi - \frac{\kappa}{\sqrt{2}} \bar{\lambda}_\mu \bar{\sigma}^{\mu\nu} \bar{\psi} \nabla_\nu \phi + \frac{\kappa}{\sqrt{2}} \lambda_\mu \sigma^{\mu\nu} \nabla_\nu \psi \phi^\dagger - \frac{\kappa}{\sqrt{2}} \lambda_\mu \sigma^{\mu\nu} \psi \nabla_\nu \phi^\dagger \\ & + \frac{i\kappa^3}{\sqrt{2}} (\bar{\lambda}_\mu \bar{\sigma}^{\mu\nu} \bar{\lambda}_\nu \lambda_\rho \sigma^\rho \bar{\psi} + 2\bar{\lambda}_\nu \bar{\sigma}^{\rho\mu} \bar{\lambda}_\rho \lambda_\mu \sigma^\nu \bar{\psi}) \phi \\ & - \frac{i\kappa^3}{\sqrt{2}} (\lambda_\mu \sigma^{\mu\nu} \lambda_\nu \psi \sigma^\rho \bar{\lambda}_\rho + 2\lambda_\nu \sigma^{\rho\mu} \lambda_\rho \psi \sigma^\nu \bar{\lambda}_\mu) \phi^\dagger \Bigg], \end{aligned} \quad (20)$$

$$\begin{aligned} \mathcal{S}_{\text{can}}^{\prime\text{NL}} = \int d^4x \det T \Bigg[& F^\dagger F + \frac{1}{2} \phi \nabla^\mu \phi_\mu^\dagger + \frac{1}{2} \phi^\dagger \nabla^\mu \phi_\mu \\ & + \frac{i}{2} \nabla_\mu \bar{\psi} \bar{\sigma}^\mu \psi - \frac{i}{2} \bar{\psi} \bar{\sigma}^\mu \nabla_\mu \psi + \frac{\kappa}{\sqrt{2}} \phi_\mu^\dagger \lambda_\nu \sigma^\mu \bar{\sigma}^\nu \psi + \frac{\kappa}{\sqrt{2}} \phi_\mu \bar{\lambda}_\nu \bar{\sigma}^\mu \sigma^\nu \bar{\psi} \\ & - 2i\kappa^2 \phi \phi_\mu^\dagger \lambda_\nu \sigma^\mu \bar{\sigma}^{\nu\rho} \bar{\lambda}_\rho + 2i\kappa^2 \phi^\dagger \phi_\mu \lambda_\rho \sigma^{\rho\nu} \sigma^\mu \bar{\lambda}_\nu \\ & + \sqrt{2}\kappa \bar{\lambda}_\mu \bar{\sigma}^{\mu\nu} \nabla_\nu \bar{\psi} \phi + \sqrt{2}\kappa \lambda_\mu \sigma^{\mu\nu} \nabla_\nu \psi \phi^\dagger \Bigg], \end{aligned} \quad (21)$$

where $\phi_\mu = \nabla_\mu \phi + \sqrt{2}\kappa\psi\lambda_\mu$. Taking $\frac{1}{4}\phi\phi^{\mu\dagger}$ as a composite field and using Eq (18), one has

$$\int d^4x \det T \left[\frac{1}{4} \nabla_\mu (\phi \phi^{\mu\dagger}) \right] = \int d^4x \det T \left[-\frac{i\kappa^2}{2} \phi \phi^{\mu\dagger} (\lambda_\mu \sigma^\nu \bar{\lambda}_\nu - \lambda_\nu \sigma^\nu \bar{\lambda}_\mu) \right]. \quad (22)$$

Taking $\frac{i\kappa}{4\sqrt{2}}\phi(\sigma^\mu \bar{\psi})_\alpha$ as a composite fermion field, Eq (17) leads to

$$\begin{aligned} & \int d^4x \det T \left[\frac{i\kappa^3}{\sqrt{2}} \phi (\bar{\lambda}_\rho \bar{\sigma}^{\nu\rho} \bar{\lambda}_\nu \lambda_\mu \sigma^\mu \bar{\psi} + 2\bar{\lambda}_\mu \bar{\sigma}^{\nu\rho} \bar{\lambda}_\rho \lambda_\nu \sigma^\mu \bar{\psi}) \right] \\ & = \int d^4x \det T \left[-\frac{\kappa}{4\sqrt{2}} \bar{\lambda}_\nu \bar{\sigma}^\nu \nabla_\mu (\sigma^\mu \bar{\psi} \phi) + \frac{\kappa}{4\sqrt{2}} \bar{\lambda}_\mu \bar{\sigma}^\nu \nabla_\nu (\sigma^\mu \bar{\psi} \phi) \right]. \end{aligned} \quad (23)$$

Substituting Eq (22), Eq (23) and their conjugate identities back into $\mathcal{S}_{\text{can}}^{\text{NL}}$, we can obtain $\mathcal{S}_{\text{can}}^{\prime\text{NL}}$, as expected. The difference between $\mathcal{S}_{\text{can}}^{\text{NL}}$ and $\mathcal{S}_{\text{can}}^{\prime\text{NL}}$ are indeed integrals over total divergences.

When coupled to gauge fields, the canonical Kahler potential in linear theory becomes $\mathcal{S}_{\text{can}}^{\text{V}} = \int d^4x d^4\theta \hat{\Phi}^\dagger e^{2g\hat{V}} \hat{\Phi}$, where \hat{V} is a vector superfield. The corresponding potential in the standard nonlinear realization of SUSY is [4]

$$\mathcal{S}_{\text{can}}^{\text{V,NL}} = \int d^4x d^4\theta \det T \det M \Phi^\dagger e^{2gV} \Phi. \quad (24)$$

Here V could be constructed from \hat{V} by Eq (4). \hat{V} and V cannot be put into the Wess-Zumino gauge simultaneously. In this paper, V is chosen to be in Wess-Zumino gauge [8]. That is to say,

$$\begin{aligned} V = & -\theta\sigma^\mu\bar{\theta}v_\mu + i\theta^2\bar{\theta}\left(\bar{\chi} - \frac{1}{2}\kappa\bar{\sigma}^\mu\sigma^\nu\bar{\lambda}_\mu v_\nu\right) - i\bar{\theta}^2\theta\left(\chi - \frac{1}{2}\kappa\sigma^\mu\bar{\sigma}^\nu\lambda_\mu v_\nu\right) \\ & + \frac{1}{2}\theta^2\bar{\theta}^2\left(D - \kappa^2\lambda_\mu\sigma^\nu\bar{\lambda}^\mu v_\nu\right). \end{aligned} \quad (25)$$

Ref. [8] tells us that this nonlinear action could be given in another form as

$$\mathcal{S}_{\text{can}}^{\text{V,NL}} = \int d^4x d^4\theta \frac{1}{2} \det T \left(\det M_+ e^{-2i\theta\sigma^\mu\bar{\theta}\Delta_\mu^-} \varphi^\dagger e^{2gU} \varphi + \det M_- \varphi^\dagger e^{2gU} e^{2i\theta\sigma^\mu\bar{\theta}\Delta_\mu^+} \varphi \right), \quad (26)$$

where $U = \exp(-i\theta\sigma^\mu\bar{\theta}\Delta_\mu)V$. $\mathcal{S}_{\text{can}}^{\text{V,NL}}$ can be shown to be equivalent to $\mathcal{S}_{\text{can}}^{\text{V,NL}}$ with the help of Eq (22), Eq (23) and their conjugate identities, following the same step of identifying $\mathcal{S}_{\text{can}}^{\text{NL}}$ with $\mathcal{S}_{\text{can}}^{\text{NL}}$.

Now we turn to a general Kahler potential. Without gauge couplings, a general Kahler potential in linear SUSY theory can be written as $\mathcal{S}_{\text{gen}} = \int d^4x d^4\theta \mathcal{K}(\hat{\Phi}^\dagger, \hat{\Phi})$ or a different form $\mathcal{S}'_{\text{gen}} = \int d^4x d^4\theta \frac{1}{2} \left[\mathcal{K}(e^{-2i\theta\sigma^\mu\bar{\theta}\partial_\mu} \hat{\Phi}^\dagger, \hat{\Phi}) + \mathcal{K}(\hat{\Phi}^\dagger, e^{2i\theta\sigma^\mu\bar{\theta}\partial_\mu} \hat{\Phi}) \right]$ up to total divergence terms. For \mathcal{S}_{gen} , its nonlinear version is [8]

$$\mathcal{S}_{\text{gen}}^{\text{NL}} = \int d^4x d^4\theta \det T \det M \mathcal{K}(\Phi^\dagger, \Phi), \quad (27)$$

Taylor expanding $\mathcal{K}(\Phi^\dagger, \Phi)$ in terms of θ and $\bar{\theta}$ and then using the substitution rules discussed

in [8], the nonlinear action can be obtained exactly as

$$\begin{aligned}
\mathcal{S}_{\text{gen}}^{\text{NL}} = \int d^4x d^4\theta \det T \Bigg\{ & \frac{1}{4} \nabla^\mu \phi_{i\mu} \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i} + \frac{1}{4} \nabla^\mu \phi_{i\mu}^\dagger \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger} + \frac{1}{2} i \kappa^2 \lambda^\mu \sigma^\nu \bar{\lambda}_\mu \phi_{i\nu} \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i} \\
& - \frac{1}{2} i \kappa^2 \lambda^\mu \sigma^\nu \bar{\lambda}_\mu \phi_{i\nu}^\dagger \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger} + \frac{\sqrt{2}}{2} \kappa \lambda_\mu \sigma^{\mu\nu} \nabla_\nu \psi_i \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i} + \frac{\sqrt{2}}{2} \kappa \bar{\lambda}_\mu \bar{\sigma}^{\mu\nu} \nabla_\nu \bar{\psi}_i \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger} \\
& + \frac{1}{2} i \kappa^2 \lambda_\nu \sigma^\nu \bar{\sigma}^\mu \sigma^\rho \bar{\lambda}_\mu \phi_{i\rho} \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i} + \frac{1}{2} i \kappa^2 \bar{\lambda}_\nu \bar{\sigma}^\nu \sigma^\mu \bar{\sigma}^\rho \lambda_\mu \phi_{i\rho}^\dagger \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger} \\
& - \frac{1}{2} \kappa^2 \epsilon^{\mu\nu\rho\gamma} \lambda_\mu \sigma_\gamma \bar{\lambda}_\nu \phi_{i\rho} \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i} + \frac{1}{2} \kappa^2 \epsilon^{\mu\nu\rho\gamma} \lambda_\mu \sigma_\gamma \bar{\lambda}_\nu \phi_{i\rho}^\dagger \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger} \\
& - \frac{\sqrt{2}}{2} i \kappa^3 \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i} \psi_i [\sigma^\rho \bar{\lambda}_\rho \lambda_\mu \sigma^{\mu\nu} \lambda_\nu + 2 \sigma^\nu \bar{\lambda}_\mu \lambda_\nu \sigma^{\rho\mu} \lambda_\rho] \\
& - \frac{\sqrt{2}}{2} i \kappa^3 \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger} \bar{\psi}_i [\bar{\sigma}^\rho \lambda_\rho \bar{\lambda}_\mu \bar{\sigma}^{\mu\nu} \bar{\lambda}_\nu + 2 \bar{\sigma}^\nu \lambda_\mu \bar{\lambda}_\nu \bar{\sigma}^{\rho\mu} \bar{\lambda}_\rho] \\
& + \frac{1}{2} i \nabla_\mu \bar{\psi}_i \bar{\sigma}^\mu \psi_j \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger \partial \phi_j} + \frac{\sqrt{2}}{2} \kappa \lambda_\mu \sigma^\nu \bar{\sigma}^\mu \psi_j \phi_{i\nu}^\dagger \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_j \partial \phi_i^\dagger} + \frac{\sqrt{2}}{2} \kappa \bar{\lambda}_\mu \bar{\sigma}^\nu \sigma^\mu \bar{\psi}_j \phi_{i\nu} \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_j^\dagger} \\
& - \frac{1}{2} i \bar{\psi}_i \bar{\sigma}^\mu \nabla_\mu \psi_j \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger \partial \phi_j} - \frac{\sqrt{2}}{4} \kappa \lambda_\mu \sigma^\mu \bar{\sigma}^\nu \psi_j \phi_{i\nu}^\dagger \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_j \partial \phi_i^\dagger} - \frac{\sqrt{2}}{4} \kappa \bar{\lambda}_\mu \bar{\sigma}^\mu \sigma^\nu \bar{\psi}_j \phi_{i\nu} \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_j^\dagger} \\
& + \frac{1}{2} \kappa^2 \epsilon^{\mu\nu\rho\gamma} \lambda_\mu \sigma_\gamma \bar{\lambda}_\nu \bar{\psi}_j \bar{\sigma}_\rho \psi_i \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_j^\dagger} + F_i^\dagger F_j \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger \partial \phi_j} - \frac{1}{2} \phi_{i\mu}^\dagger \phi_j^\mu \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger \partial \phi_j} \\
& + \frac{1}{4} \phi_{i\mu} \phi_j^\mu \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_j} + \frac{1}{2} \kappa^2 \lambda_\mu \sigma^{\mu\nu} \lambda_\nu \psi_i \psi_j \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_j} + \frac{1}{2} \kappa^2 \bar{\lambda}_\mu \bar{\sigma}^{\mu\nu} \bar{\lambda}_\nu \bar{\psi}_i \bar{\psi}_j \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger \partial \phi_j^\dagger} \\
& + \frac{1}{4} \phi_{i\mu}^\dagger \phi_j^\mu \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger \partial \phi_j^\dagger} + \frac{\sqrt{2}}{4} \kappa \lambda_\nu \sigma^\nu \bar{\sigma}^\mu \psi_j \phi_{i\mu} \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_j} + \frac{\sqrt{2}}{4} \kappa \bar{\lambda}_\nu \bar{\sigma}^\nu \sigma^\mu \bar{\psi}_j \phi_{i\mu}^\dagger \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger \partial \phi_j^\dagger} \\
& + \frac{1}{2} i \psi_k \sigma^\mu \bar{\psi}_i \phi_{l\mu} \frac{\partial^3 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger \partial \phi_k \partial \phi_l} - \frac{1}{2} i \psi_i \sigma^\mu \bar{\psi}_k \phi_{l\mu}^\dagger \frac{\partial^3 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_k^\dagger \partial \phi_l^\dagger} \\
& + \frac{\sqrt{2}}{4} i \kappa \lambda_\mu \sigma^\mu \bar{\psi}_i \psi_k \psi_l \frac{\partial^3 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger \partial \phi_k \partial \phi_l} - \frac{\sqrt{2}}{4} i \kappa \psi_i \sigma^\mu \bar{\lambda}_\mu \bar{\psi}_k \bar{\psi}_l \frac{\partial^3 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_k^\dagger \partial \phi_l^\dagger} \\
& \left. - \frac{1}{2} \bar{\psi}_k \bar{\psi}_l F_i \frac{\partial^3 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_k^\dagger \partial \phi_l^\dagger} - \frac{1}{2} \psi_i \psi_j F_k^\dagger \frac{\partial^3 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_j \partial \phi_k^\dagger} + \frac{1}{4} \psi_i \psi_j \bar{\psi}_k \bar{\psi}_l \frac{\partial^4 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_j \partial \phi_k^\dagger \partial \phi_l^\dagger} \right\}.
\end{aligned}$$

Here fields with indices in Latin letters i, j, k, l stand for different components while fields with indices in Greek letters are defined as $\phi_\mu = \nabla_\mu \phi + \sqrt{2} \kappa \psi \lambda_\mu$.

For $\mathcal{S}'_{\text{gen}}$, its nonlinear version is [8]

$$\mathcal{S}_{\text{gen}}^{\text{NL}} = \int d^4x d^4\theta \frac{1}{2} \det T \left[\det M_+ \mathcal{K}(e^{-2i\theta\sigma^\mu \bar{\theta} \Delta_\mu^-} \varphi^\dagger, \varphi) + \det M_- \mathcal{K}(\varphi^\dagger, e^{2i\theta\sigma^\mu \bar{\theta} \Delta_\mu^+} \varphi) \right]. \quad (28)$$

After expanding, one has

$$\begin{aligned}
\mathcal{S}'_{\text{gen}}^{\text{NL}} = \int d^4x d^4\theta \det T \Bigg\{ & \frac{1}{2} \nabla^\mu \phi_{k\mu} \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k} + \frac{1}{2} \nabla^\mu \phi_{k\mu}^\dagger \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k^\dagger} \\
& + i\kappa^2 \lambda^\mu \sigma^\nu \bar{\lambda}_\mu \phi_{k\nu} \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k} - i\kappa^2 \lambda^\mu \sigma^\nu \bar{\lambda}_\mu \phi_{k\nu}^\dagger \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k^\dagger} \\
& + \sqrt{2} \kappa \lambda_\mu \sigma^{\mu\nu} \nabla_\nu \psi_k \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k} + \sqrt{2} \kappa \bar{\lambda}_\mu \bar{\sigma}^{\mu\nu} \nabla_\nu \bar{\psi}_k \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k^\dagger} \\
& + i\kappa^2 \lambda_\mu \sigma^\mu \bar{\sigma}^\nu \sigma^\rho \bar{\lambda}_\nu \phi_{k\rho} \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k} + i\kappa^2 \bar{\lambda}_\mu \bar{\sigma}^\mu \sigma^\nu \bar{\sigma}^\rho \lambda_\nu \phi_{k\rho}^\dagger \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k^\dagger} \\
& + \frac{1}{2} i \nabla_\mu \psi_k \sigma^\mu \bar{\psi}_i \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k \partial \phi_i^\dagger} - \frac{1}{2} i \psi_i \sigma^\mu \nabla_\mu \bar{\psi}_k \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_k^\dagger} + F_i F_k^\dagger \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_k^\dagger} \\
& + \frac{\sqrt{2}}{2} \kappa \phi_\nu^\dagger \lambda_\mu \sigma^\nu \bar{\sigma}^\mu \psi_i \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_k^\dagger} + \frac{\sqrt{2}}{2} \kappa \phi_\nu \bar{\lambda}_\mu \bar{\sigma}^\nu \sigma^\mu \bar{\psi}_i \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k \partial \phi_i^\dagger} \\
& + \frac{1}{2} \phi_{k\mu} \phi_l^\mu \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k \partial \phi_l} + \frac{1}{2} \phi_{k\mu}^\dagger \phi_l^{\dagger\mu} \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k^\dagger \partial \phi_l^\dagger} \\
& + \kappa^2 \lambda_\mu \sigma^{\mu\nu} \lambda_\nu \psi_k \psi_l \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k \partial \phi_l} + \kappa^2 \bar{\lambda}_\mu \bar{\sigma}^{\mu\nu} \bar{\lambda}_\nu \bar{\psi}_k \bar{\psi}_l \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k^\dagger \partial \phi_l^\dagger} \\
& + \frac{\sqrt{2}}{2} \kappa \lambda_\mu \sigma^\mu \bar{\sigma}^\nu \psi_k \phi_{l\nu} \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k \partial \phi_l} + \frac{\sqrt{2}}{2} \kappa \bar{\lambda}_\mu \bar{\sigma}^\mu \sigma^\nu \bar{\psi}_k \phi_{l\nu}^\dagger \frac{\partial^2 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_k^\dagger \partial \phi_l^\dagger} \\
& + \frac{\sqrt{2}}{4} i \kappa \lambda_\mu \sigma^\mu \bar{\psi}_i \psi_k \psi_l \frac{\partial^3 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger \partial \phi_k \partial \phi_l} - \frac{\sqrt{2}}{4} i \kappa \psi_i \sigma^\mu \bar{\lambda}_\mu \bar{\psi}_k \bar{\psi}_l \frac{\partial^3 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_k^\dagger \partial \phi_l^\dagger} \\
& + \frac{1}{2} i \psi_k \sigma^\mu \bar{\psi}_i \phi_{l\mu} \frac{\partial^3 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger \partial \phi_k \partial \phi_l} - \frac{1}{2} i \psi_i \sigma^\mu \bar{\psi}_k \phi_{l\mu}^\dagger \frac{\partial^3 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_k^\dagger \partial \phi_l^\dagger} \\
& - \frac{1}{2} \bar{\psi}_k \bar{\psi}_l F_i \frac{\partial^3 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_k^\dagger \partial \phi_l^\dagger} - \frac{1}{2} \psi_i \psi_j F_k^\dagger \frac{\partial^3 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_j \partial \phi_k^\dagger} + \frac{1}{4} \psi_i \psi_j \bar{\psi}_k \bar{\psi}_l \frac{\partial^4 \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i \partial \phi_j \partial \phi_k^\dagger \partial \phi_l^\dagger} \Bigg\}.
\end{aligned}$$

$\mathcal{S}_{\text{gen}}^{\text{NL}}$ and $\mathcal{S}'_{\text{gen}}^{\text{NL}}$ should describe the same physical process in the nonlinear SUSY realization. We utilize our new identities to find out the nonlinear total-divergence terms. Taking $\frac{1}{4} \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger} \phi^{\mu\dagger}$ as a composite field into Eq (18), one has

$$\int d^4x \det T \frac{1}{4} \nabla_\mu \left[\frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger} \phi_i^{\mu\dagger} \right] = \int d^4x \det T \left[-\frac{i\kappa^2}{2} \frac{\partial \mathcal{K}(\phi^\dagger, \phi)}{\partial \phi_i^\dagger} \phi_i^{\mu\dagger} (\lambda_\mu \sigma^\nu \bar{\lambda}_\nu - \lambda_\nu \sigma^\nu \bar{\lambda}_\mu) \right]. \quad (29)$$

Taking $\frac{\sqrt{2}i\kappa}{8}(\sigma^\mu\bar{\psi}_i)_\alpha\frac{\partial\mathcal{K}(\phi^\dagger,\phi)}{\partial\phi_i^\dagger}$ as a composite fermion field, Eq (17) gives

$$\begin{aligned} & \int d^4x \det T \left[\frac{i\kappa^3}{\sqrt{2}} \frac{\partial\mathcal{K}(\phi^\dagger,\phi)}{\partial\phi_i^\dagger} (\bar{\lambda}_\rho\bar{\sigma}^{\nu\rho}\bar{\lambda}_\nu\lambda_\mu\sigma^\mu\bar{\psi} + 2\bar{\lambda}_\mu\bar{\sigma}^{\nu\rho}\bar{\lambda}_\rho\lambda_\nu\sigma^\mu\bar{\psi}) \right] \\ &= \int d^4x \det T \left\{ -\frac{\kappa}{4\sqrt{2}}\bar{\lambda}_\nu\bar{\sigma}^\nu\nabla_\mu \left[\sigma^\mu\bar{\psi} \frac{\partial\mathcal{K}(\phi^\dagger,\phi)}{\partial\phi_i^\dagger} \right] + \frac{\kappa}{4\sqrt{2}}\bar{\lambda}_\mu\bar{\sigma}^\mu\nabla_\nu \left[\sigma^\nu\bar{\psi} \frac{\partial\mathcal{K}(\phi^\dagger,\phi)}{\partial\phi_i^\dagger} \right] \right\}. \end{aligned} \quad (30)$$

Substituting Eq (29), Eq (30) and their conjugate identities back into $\mathcal{S}_{\text{gen}}^{\text{NL}}$, we obtain $\mathcal{S}_{\text{gen}}^{\text{NL}}$ from $\mathcal{S}_{\text{gen}}^{\text{NL}}$. For a general Kahler potential coupled to a gauge field, the situation is similar. The proof is still based on Eq (29), Eq (30) and their conjugate identities. Moreover, Eq (29) and Eq (30) can be exactly identical to Eq (22) and Eq (23) if let $\mathcal{K}(\hat{\Phi}^\dagger, \hat{\Phi}) = \hat{\Phi}^\dagger\hat{\Phi}$.

Besides the standard nonlinear realization, there are already other formalisms that realize SUSY algebra nonlinearly. Keeping the dual description in mind, the total-divergence terms can be easily obtained in these formalisms. We take the chiral nonlinear realization as an example. In chiral nonlinear realization, the explicit SUSY transformations of the Goldstino field $\tilde{\lambda}$ and matter fields $\tilde{\zeta}$ are [7], [16], [17]

$$\begin{cases} \delta_\xi\tilde{\lambda}_\alpha &= \frac{\xi_\alpha}{\kappa} - 2i\kappa\tilde{\lambda}\sigma^\mu\bar{\xi}\partial_\mu\tilde{\lambda}_\alpha, \\ \delta_\xi\tilde{\zeta} &= -2i\kappa\tilde{\lambda}\sigma^\mu\bar{\xi}\partial_\mu\tilde{\zeta}. \end{cases} \quad (31)$$

In principle, both linear realization and this chiral nonlinear realization can form dual descriptions of the same UV SUSY-breaking theory. The total-derivatives in this chiral nonlinear formalism could be obtained from ones in the linear realization. But such a construction is not needed in practice, since it is well-known that the chiral fields $\tilde{\lambda}$ and $\tilde{\zeta}$ can be constructed from λ and ζ in terms of

$$\tilde{\lambda}_\alpha(x) = \lambda_\alpha(\omega), \quad \tilde{\zeta}(x) = \zeta(\omega), \quad (32)$$

with $\omega = x - i\kappa^2\lambda(\omega)\sigma\bar{\lambda}(\omega)$. So all the identities about λ , ζ can be converted into identities about $\tilde{\lambda}$, $\tilde{\zeta}$ by changing of variables and adding the Jacobian determinant of the transformation. This means that different nonlinear realizations could form dual descriptions of the same theory. For other nonlinear realizations, explicit maps between them and the standard nonlinear realization has been exactly given in [18]. After changes of variables, the total-derivatives in these nonlinear formalisms could be obtained from ones in the standard nonlinear realization. Finally, we pay attention to the constrained superfield formalism,

which has been proposed to construct nonlinear fields and actions [19], [20], [21]. The constrained superfield formalism could be regarded as a regular linear realization with some constraints. The total-derivatives in the constrained formalism are the same as ordinary linear ones. The constraints only require that some component fields are not fundamental, but consist of other component fields.

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Appendix A: More Identities in Standard Nonlinear SUSY Realization

For a general super-field $\hat{\Phi} = \hat{\phi} + \theta \hat{f} + \bar{\theta} \hat{g} + \hat{m} \theta^2 + \hat{n} \bar{\theta}^2 + \theta \sigma^\delta \bar{\theta} \hat{\nu}_\delta + \bar{\theta}^2 \theta \hat{\chi} + \theta^2 \bar{\theta} \hat{\psi} + \hat{D} \theta^2 \bar{\theta}^2$ in linear theory, we always have $\int d^4 x d^4 \theta \partial_\mu \hat{\Phi} = 0$. In terms of nonlinear fields, one has

$$\int d^4 x d^4 \theta \det T \det M \Delta_\mu \Phi = 0, \quad (\text{A1})$$

where $\Delta_\mu = (M^{-1})_\mu^\nu (\nabla_\nu + \kappa \lambda_\nu^\alpha \partial_\alpha + \kappa \bar{\lambda}_{\nu\dot{\alpha}} \partial^{\dot{\alpha}})$ and $\Phi = \phi + \theta f + \bar{\theta} \bar{g} + m \theta^2 + n \bar{\theta}^2 + \theta \sigma^\delta \bar{\theta} \nu_\delta + \bar{\theta}^2 \theta \chi + \theta^2 \bar{\theta} \psi + D \theta^2 \bar{\theta}^2$. The matter fields in Φ are composites of component fields in $\hat{\Phi}$ and the Goldstino field λ . Their explicit forms are governed by Eq (4).

Defining

$$\det M (M^{-1})_\mu^\nu = \delta_\mu^\nu + \theta \psi_{M\mu}^\nu + \bar{\theta} \bar{\psi}_{M\mu}^\nu + \theta \sigma_\gamma \bar{\theta} \nu_{M\mu}^{\gamma\nu} + \theta^2 F_{M\mu}^\nu + \bar{\theta}^2 F_{M\mu}^{\nu\dagger} + \theta^2 \bar{\theta} \bar{\chi}_{M\mu}^\nu + \bar{\theta}^2 \theta \chi_{M\mu}^\nu + \theta^2 \bar{\theta}^2 D_{M\mu}^\nu,$$

where

$$\begin{aligned} \psi_{M\mu\alpha}^\nu &= i\kappa \left[(\sigma^\nu \bar{\lambda}_\mu)_\alpha - (\sigma^\rho \bar{\lambda}_\rho)_\alpha \delta_\mu^\nu \right], \\ \nu_{M\mu}^{\gamma\nu} &= i\kappa^2 \left[\epsilon^{\nu\gamma\rho\delta} (\lambda_\rho \sigma_\delta \bar{\lambda}_\mu - \lambda_\mu \sigma_\delta \bar{\lambda}_\rho) + \epsilon^{\gamma\sigma\rho\delta} \lambda_\rho \sigma_\delta \bar{\lambda}_\sigma \delta_\mu^\nu \right], \\ F_{M\mu}^\nu &= \kappa^2 \left[2 \bar{\lambda}_\mu \bar{\sigma}^{\nu\rho} \bar{\lambda}_\rho - \bar{\lambda}_\rho \bar{\sigma}^{\rho\sigma} \bar{\lambda}_\sigma \delta_\mu^\nu \right], \\ \chi_{M\mu\alpha}^\nu &= \frac{i}{2} \kappa^3 \left[2 (\sigma^\rho \bar{\lambda}_\rho)_\alpha \lambda_\sigma \sigma^{\sigma\gamma} \lambda_\gamma \delta_\mu^\nu + 4 (\sigma^\gamma \bar{\lambda}_\sigma)_\alpha \lambda_\gamma \sigma^{\rho\sigma} \lambda_\rho \delta_\mu^\nu \right. \\ &\quad - 2 (\sigma^\nu \bar{\lambda}_\mu)_\alpha \lambda_\rho \sigma^{\rho\sigma} \lambda_\sigma + 4 (\sigma^\rho \bar{\lambda}_\mu)_\alpha \lambda_\rho \sigma^{\nu\sigma} \lambda_\sigma + 4 (\sigma^\nu \bar{\lambda}_\rho)_\alpha \lambda_\mu \sigma^{\rho\sigma} \lambda_\sigma \\ &\quad \left. + (\sigma^\sigma \bar{\lambda}_\sigma)_\alpha \lambda_\mu \sigma^\rho \bar{\sigma}^\nu \lambda_\rho - (\sigma^\sigma \bar{\lambda}_\rho)_\alpha \lambda_\mu \sigma^\rho \bar{\sigma}^\nu \lambda_\sigma + i \epsilon^{\gamma\sigma\rho\delta} (\sigma_\delta \bar{\sigma}^\nu \lambda_\mu)_\alpha \lambda_\rho \sigma_\gamma \bar{\lambda}_\sigma \right], \end{aligned}$$

$$\begin{aligned}
D_{M\mu}^\nu = & \frac{1}{4}\kappa^4 \left[4\lambda_\sigma\sigma^{\gamma\nu}\lambda_\gamma\bar{\lambda}_\rho\bar{\sigma}^\sigma\sigma^\rho\bar{\lambda}_\mu + 4\lambda_\mu\sigma^\rho\bar{\sigma}^\sigma\lambda_\rho\bar{\lambda}_\gamma\bar{\sigma}^{\nu\gamma}\bar{\lambda}_\sigma \right. \\
& - 4\lambda_\mu\sigma^\nu\bar{\sigma}^\gamma\lambda_\sigma\bar{\lambda}_\gamma\bar{\sigma}^{\rho\sigma}\bar{\lambda}_\rho - 4\bar{\lambda}_\sigma\bar{\sigma}^\gamma\sigma^\nu\bar{\lambda}_\mu\lambda_\gamma\sigma^{\rho\sigma}\lambda_\rho \\
& + 8\lambda_\sigma\sigma^{\sigma\gamma}\lambda_\gamma\bar{\lambda}_\mu\bar{\sigma}^{\rho\nu}\bar{\lambda}_\rho + 8\bar{\lambda}_\sigma\bar{\sigma}^{\sigma\gamma}\bar{\lambda}_\gamma\lambda_\mu\sigma^{\rho\nu}\lambda_\rho \\
& + \lambda_\rho\sigma^\sigma\bar{\sigma}^\nu(\lambda_\gamma\bar{\lambda}_\sigma - \lambda_\sigma\bar{\lambda}_\gamma)\bar{\sigma}^\gamma\sigma^\rho\bar{\lambda}_\mu + \lambda_\mu\sigma^\rho\bar{\sigma}^\gamma(\lambda_\sigma\bar{\lambda}_\gamma - \lambda_\gamma\bar{\lambda}_\sigma)\bar{\sigma}^\nu\sigma^\sigma\bar{\lambda}_\rho \\
& + \lambda_\mu\sigma^\rho\bar{\sigma}^\nu(\lambda_\gamma\bar{\lambda}_\sigma - \lambda_\sigma\bar{\lambda}_\gamma)\bar{\sigma}^\gamma\sigma^\sigma\bar{\lambda}_\rho + \lambda_\rho\sigma^\sigma\bar{\sigma}^\gamma(\lambda_\sigma\bar{\lambda}_\gamma - \lambda_\gamma\bar{\lambda}_\sigma)\bar{\sigma}^\nu\sigma^\rho\bar{\lambda}_\mu \\
& \left. + i(\lambda_\mu\sigma^\rho\bar{\sigma}_\delta\sigma^\nu\bar{\lambda}_\rho + \lambda_\rho\sigma^\nu\bar{\sigma}_\delta\sigma^\rho\bar{\lambda}_\mu)\epsilon^{\sigma\gamma\delta\epsilon}\lambda_\sigma\sigma_\epsilon\bar{\lambda}_\gamma \right].
\end{aligned}$$

Being a matter field in the nonlinear realization, each component field of Φ transforms into itself and is independent of other fields. Thus Eq (A1) contains nine independent identities.

They are:

$$\int d^4x \det T \left[\nabla_\mu D + 2i\kappa^2 D(\lambda_\mu\sigma^\nu\bar{\lambda}_\nu - \lambda_\nu\sigma^\nu\bar{\lambda}_\mu) \right] = 0, \quad (\text{A2})$$

$$\int d^4x \det T \left(\kappa F_{M\mu}^{\nu+} \bar{\lambda}_\nu \bar{\psi} - \frac{1}{2} \bar{\psi}_{M\mu}^\nu \nabla_\nu \bar{\psi} + \frac{1}{2} \kappa v_{M\mu}^{\gamma\nu} \lambda_\nu \sigma_\gamma \bar{\psi} \right) = 0, \quad (\text{A3})$$

$$\int d^4x \det T \left(\kappa F_{M\mu}^\nu \lambda_\nu \chi - \frac{1}{2} \psi_{M\mu}^\nu \nabla_\nu \chi + \frac{1}{2} \kappa v_{M\mu}^{\gamma\nu} \chi \sigma_\gamma \bar{\lambda}_\nu \right) = 0, \quad (\text{A4})$$

$$\int d^4x \det T \left[v_{M\mu}^{\delta\nu} \nabla_\nu \nu_\delta + \kappa \nu_\delta (\lambda_\nu \sigma^\delta \bar{\chi}_{M\mu}^\nu + \chi_{M\mu}^\nu \sigma^\delta \bar{\lambda}_\nu) \right] = 0, \quad (\text{A5})$$

$$\int d^4x \det T (F_{M\mu}^\nu \nabla_\nu n - \kappa n \bar{\chi}_{M\mu}^\nu \bar{\lambda}_\nu) = 0, \quad (\text{A6})$$

$$\int d^4x \det T (F_{M\mu}^{\nu\dagger} \nabla_\nu m - \kappa m \chi_{M\mu}^\nu \lambda_\nu) = 0, \quad (\text{A7})$$

$$\int d^4x \det T (\kappa D_{M\mu}^\nu \bar{\lambda}_\nu \bar{g} - \frac{1}{2} \bar{\chi}_{M\mu}^\nu \nabla_\nu \bar{g}) = 0, \quad (\text{A8})$$

$$\int d^4x \det T (\kappa D_{M\mu}^\nu \lambda_\nu f - \frac{1}{2} \chi_{M\mu}^\nu \nabla_\nu f) = 0, \quad (\text{A9})$$

$$\int d^4x \det T D_{M\mu}^\nu \nabla_\nu \phi = 0. \quad (\text{A10})$$

Noticing that Eq (A2) is exactly identical with Eq (18). Eq (A3), Eq (A6) and Eq (A8) are conjugate identities of Eq (A4), Eq (A7) and Eq (A9) respectively.

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